A TRADE MODEL OF INTERMEDIATE PRODUCTS WITH TRANSPORTATION COST

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This paper presents a trade model of intermediate products where a country has the same production technology as the outside world, and the source of trade is the unbalance in the factor endowment. The decision as to how much to process the raw material before exporting depends on the capital requirements for the processing, and the change in transportation cost due to the processing. The lower the requirements of capital coupled with a more rapid fall in the transportation cost in the earlier stages of production, the more probable that the country will export the processed intermediate good instead of the raw material.

I. INTRODUCTION

Almost all the classical trade models focus on the trade of final consumption goods, which are produced using factors such as labor and/or capital. However more than half of the world trade consists of intermediate products. There are only a few works that recognize this fact. Another point is that within intermediate products there may be a continuum of intermediate products with different level of processing. Some intermediate goods, such as wood and oil, are close to raw material, whereas some, such as textile and tires, are closer to final consumption good. If a country can produce tires from oil, both of which are intermediate goods, which of the two would the country export, oil or tires?

Sanyal(1983) built a model with a continuum of stages of intermediate product. The cause of trade in his model is the difference in the production technology. Some countries have comparative advantage in the earlier stage of production and others in the later. Each country specializes in some stage of production and the only way to change this specialization pattern is the change in technology. Sanyal does not explain where the difference in technology comes from, or why the technological progress has to increase the trade of intermediate goods as observed in the economic history. In Dixit and Grossman(1982), the intermediate product at a higher stage is produced from one unit of lower stage intermediate product, and capital and labor that can be substituted for each other.

There are other papers which instead of explaining the cause of trade in intermediate products, analyze other economic effects in the presence of intermediate products. Sanyal and Jones(1982), to analyze the consequences for a small trading community of changes in world
conditions, assumed a two tier model, where, in the 'Input Tier' primary productive factors and labor produce intermediate products which can be traded on world markets but are never consumed directly as final products, and in the 'Output Tier' intermediate products are combined with labor to produce final nontraded consumption goods. Mena(1992) analyzed the effects of international transfers when consumables are nontraded goods and international trade takes place in intermediate products. The stage at which an intermediate good is traded is determined mainly by factor endowment considerations.

This paper presents a model where a country has the same technology as the outside world, and the source of trade is the unbalance in the factor endowment. In this model, the difference in technology is not a necessary condition to generate trade in intermediate goods, and instead difference in the stock of production factor can explain the pattern in this type of trade. The level of processing of the exported and imported intermediate products is determined by the factor endowment together with the transportation cost.

II. MODEL

A. Without Transportation Cost

A_s is the amount of intermediate product with s level of processing (0≤s≤1). s is equal to zero for raw material and to unity for final consumption good.

The production of intermediate product (w) requires as inputs either raw material and capital, or a less processed intermediate product (v) and capital. The production function is the following Leontieff type:

\[ A_v = \min(A_v, \frac{K}{\int_v^w K(s)ds}) \]  

where \( \int_v^w K(s)ds \) is the amount of capital required to transform one unit of intermediate good of type v to one unit of type w.

The external world has the same technology. With competition, the world price of a unit of intermediate product type w is the following:

\[ P(w) = P(v) + r \int_v^w K(s)ds = P(0) + r \int_0^w K(s)ds \]  

where P(s) and r are the price of intermediate product type s and capital in the world market, respectively\(^1\).

\(^1\)In the model, there is no mechanism by which P(0) and r are determined. It can be assumed that the external world has a technology that the country does not. For example, the external world can produce the capital from the raw material:

K = qA_0
As there is only one good for consumption ($A_1$), the objective of the country is to maximize the consumption of this good.

It is assumed that the country is small with endowment $A_0$ of raw material and $K$ of capital. The endowment of capital is assumed to be insufficient to transform all the raw material endowed to final consumption good. As the country can not process all its raw material to final consumption good, the country processes its raw material to intermediate product of level $v$, export it, import a more processed intermediate good of type $w$ ($v < w$) and process it to final consumption good.

The capital is used to process the raw material to intermediate product type $v$ and to transform the imported intermediate good type $w$ to final consumption good. The full employment condition of capital is the following:

$$A_0 \int_0^1 K(s)ds + A_1 \int_0^1 K(s)ds = K$$

(3)

The equilibrium in the balance of payments implies:

$$P(v)A_v = P(w)A_w$$

(4)

The amount of $A_v$ is the same as $A_0$, and the only difference is that $A_v$ is more processed than $A_0$. In the same way, the amount of $A_w$ is the same as that of the final consumption $A_1$. The objective of the country is to import as much of intermediate good type $w$ as it can process to final consumption good.

$$A_1 = \frac{P(v)}{P(w)} A_0 = \frac{P(0) + r \int_0^1 K(s)ds}{P(1) - r \int_0^1 K(s)ds} A_0$$

(5)

Combining (3) and (5), we can obtain the following result:

In this case, the capital price will be:

$$r = qP(0)$$

The price of final consumption good will be:

$$P(1) = P(0) + qP(0) \left( \frac{1}{10} K(s)ds \right) = P(0)(1 + q \int_0^1 K(s)ds)$$

With this technology the international market would not have either excess demand or excess supply of raw material or capital.

2 The consumption in autarky state is $A_1 = \min(A_0, \frac{K}{\int_0^1 K(s)ds}) = \frac{K}{\int_0^1 K(s)ds}$

3 With a country endowed with more capital than enough to transform all its raw material to final consumption good, the argument is symmetric: the country imports intermediate goods type $v$ and exports part of the production of more processed intermediate goods (type $w$), processing the other part for domestic consumption.
A_1 = \{ \frac{P(0)A_0 + rK}{P(1)} \} (6)

This means that the final consumption will be the same regardless of the type of intermediate product the country exports or imports^4. The value of final consumption will be the value of the endowment of primary factors (raw material and capital) at world prices.

**B. With Transportation Cost**

Now, let’s assume that there is transportation cost. When transporting one unit of intermediate good of type s, there is a loss of C(s) as a fraction of total amount^5. This fraction is a decreasing function of the level of processing^6. It is also assumed that P(v)/(1-C(v)) is increasing function of v, which means that the cost (including price and transportation cost) of importing one unit of intermediate good is higher, the higher is the level of processing. If the country exports A_v and imports A_w, equilibrium in the balance of payments implies the following condition: ^7

\[
P(v)A_v(1-C(v)) = \frac{P(w)A_w}{1-C(w)} (7)
\]

The problem of the country now is to maximize the following function:

\[
A_1 = A_w = \frac{P(v)}{P(w)}(1-C(v))(1-C(w))A_v = \frac{P(v)}{P(w)}(1-C(v))(1-C(w))A_0 (8)
\]

where 0 ≤ v ≤ v^*, and v^* satisfies:

\[
A_0 \int_0^{v^*} K(s)ds = K (9)
\]

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^4 This indeterminacy of the trade pattern is eliminated when transportation cost is introduced, similar to Hong(1970).

^5 This 'iceberg' type of transportation cost is similar to that in Krugman(1980), except that the fraction here, instead of being fixed, depends on the level of processing.

^6 This pattern of transportation cost may be justified by different examples: it is less costly to transport metal plate than equivalent metal ore, frozen chicken meat than equivalent live chickens, grain of wheat than equivalent wheat plants, etc. There may be counter-examples, like timber and furniture. However, as we are dealing with a representative good the regular case may apply.

^7 The transportation cost is not symmetric for exports and imports. In the case of exports, the revenue is the price minus the transportation cost. That is:

\[
P(v)(1-C(v))=P(v)-C(v)P(v)
\]

In the case of imports, total cost is the price plus the transportation cost:

\[
P(w)/(1-C(w))=P(w)+C(w)P(w)/(1-C(w))
\]
The objective function is equivalent to the value of raw material and capital at world prices less the cost of transportation as can be seen in the Appendix A.

\[ P(1)A_1 = P(0)A_0 + rK - P(v)A_vC(v) - P(w)A_wC(w)/(1-C(w)) \]  

(10)

Differentiating with respect to \( v \):

\[
\frac{d(A_1)}{dv} = \left[ P'(v) - P'(v)C(v) - P(v)C'(v) \right] \frac{1-C(w)}{P(w)} A_0 
- \frac{P'(w) - P'(w)C(w) + P(w)C'(w)}{P(w)^2} P(v)(1-C(v)) A_0 \frac{dw}{dv}
\]

(11)

\[
\frac{d(A_1)}{dv} = \left[ \frac{P'(v)(1-C(v)) - P(v)C'(v)}{P(v)(1-C(v))} \right] - \frac{P'(w)(1-C(w)) + P(w)C'(w)}{P(w)(1-C(w))} \frac{dw}{dv}
\]

\[
\times \frac{P(v)(1-C(v))(1-C(w))}{P(w)} A_0
\]

(12)

It can be proved that \( dw/dv \) is positive (Appendix B). With some manipulation, we get the following equation:

\[
\frac{d(A_1)}{dv} = \left[ A_1 P'(v) - A_v P'(w) \frac{dw}{dv} \right] - \left[ A_v P'(v)C(v) + A_v \frac{P'(w)C(w) \frac{dw}{dv}}{1-C(w)} \right]
\]

\[
- \left[ A_v P(v)C'(v) + A_v \frac{P(w)C'(w) \frac{dw}{dv}}{(1-C(w))^2} \right] \frac{1-C(w)}{P(w)} A_0
\]

(13)

Increasing the level of processing of the exports, and as a consequence that of the imports, has three effects. The first is the increase in the export price of which the benefit tends to be offset by the increase in import price. The second is the increase in transportation cost that comes from the increase in the value transported. The third is the reduction of the unit cost of transportation.\(^8\)

i) If \( d(A_1)/dv \) is negative in \( 0 \leq v \leq v^* \), \( v \) is equal to zero. The country exports raw material and imports intermediate product and processes it to final consumption good.

ii) If \( d(A_1)/dv \) is positive in \( 0 \leq v \leq v^* \), \( v \) is equal to \( v^* \). The country exports intermediate product and imports final consumption good.

iii) If \( d(A_1)/dv \) is equal to zero for a \( v \) between 0 and \( v^* \), the country exports and imports intermediate product.

\(^8\) see footnote 7.
One example of the last case can occur when:

\[
d(A_1)/dv (v=0) > 0 \tag{14}
\]

\[
d(A_1)/dv (v=\nu^*) < 0 \tag{15}
\]

On the other hand, from the mechanism of determination of prices, we get:

\[
P'(s) = rK(s) \tag{16}
\]

Substituting (16) in (13), we obtain:

\[
\frac{d(A_1)}{dv} = \left[ A_rK(v) - A_rK(w) \frac{dw}{dv} \right] - \left[ A_rK(v)C(v) + A_p \frac{rK(w)C(w) \frac{dw}{dv}}{1 - C(w)} \right] - \left[ A_pP(v)C'(v) + \frac{A_pP(w)C'(w) \frac{dw}{dv}}{(1 - C(w))^2} \right] \frac{1 - C(w)}{P(w)} \tag{17}
\]

As explained before, the effects of the last two terms dominate. Therefore, it is more probable that the internal solution occur when:

i) The requirement of capital is lower in the earlier stages of production or at the stage of \( w(v=0) \), and higher in the later stages or at \( \nu^* \).

ii) The transportation cost falls rapidly in the earlier stages or at the stage of \( w(v=0) \), and slowly in the later stages or at \( \nu^* \).

![Figure 1. K(s)](image-url)
For example, we can consider a case where $K(s)$ is an increasing function and $C(s)$ a decreasing function of $s$ as illustrated in the figures 1 and 2. With the amount of capital available in the country, the country can export all its raw material and import intermediate product type $w(0)$ to process it into a final consumption good with its capital. In this case, it must pay high transportation cost for the export of its raw material. The country can benefit by processing its exports to a higher level. Doing so, the country can reduce its unit transportation cost drastically, whereas the value added that must pay the transportation cost does not increase as much.

Nevertheless, the country would not process its exports to the highest level, which is $v^*$. If it exports intermediate good type $v^*$, it would have to import final consumption good because it already would have used all its capital to process its exports. As the unit transportation cost is already low, the increase in value added would increase the transportation cost of its exports and especially imports. In equilibrium, the country would export intermediate product with a level of processing between 0 and $v^*$ and import one between $w(0)$ and 1.\(^9\)

Another interesting case is when $C(s) = C$. The transportation cost as a proportion of the volume is constant. In this case, $d(A_1)/dv$ is negative for $0 \leq v \leq v^*$, and the optimal value of $v$ is zero. This is because if some value is added to $A_0$, this value added also has to pay the transportation cost, as can be seen in the following equation:

$$P(1)A_1 = P(0)A_0 + rK - P(v)A_v2C(1-C)$$

$$= P(0)A_0 + rK - \left[ P(0) + \int_0^v K(s)ds \right] A_0 C(2-C)$$  \hspace{1cm} (18)

\(^9\) If $C(s)$ is non-monotonic, it would be possible to have a case where the country exports (and imports) more than one type of intermediate products.
If \( v \) is equal to zero, the final consumption is the following:

\[
P(1)A_1 = P(0)A_0 + rK - P(0)A_02C(1-C)
\]  

(19)

**C. Partial Trade**

In the previous model, all the raw material was processed to the same level and exported, and all the final good was made from the imported intermediate good. As all the raw material is homogeneous, one would expect that all of it would be treated in the same way. That is, processed to the same level and exported. But this is not always true. In this section, we allow for a fraction of the raw material to be exported after being processed to a certain level, and the other to be processed to the final stage and consumed internally.

For an illustration, let’s consider the case where \( C(s)=C \). If total capital is used to process as much of raw material as possible to the final level, and the rest of the raw material is exported to import final consumption good with the proceeds, the final consumption would be:

\[
P(1)A_1 = P(0)A_0 + rK - P(0)X_0C - P(1)M_1C/(1-C)
\]  

(20)

where \( X_s \) and \( M_s \) are export and import of the intermediate product type \( s \), respectively. The amount exported is equal to the raw material endowed less those processed to the final stage.

\[
X_0 = A_0 - \int_0^1 K(s)ds
\]  

(21)

The equilibrium in the balance of payments implies:

\[
P(0)X_0(1-C) = P(1)M_1/(1-C)
\]  

(22)

Substituting this condition to the equation, we obtain:

\[
P(1)A_1 = P(0)A_0 + rK - P(0)X_0C(2-C)
\]  

(23)

As \( X_0 \) is less than \( A_0 \), this strategy gives more consumption than the equation (19). Consumption is larger now because a fraction of value added does not pay the transportation cost that had to be paid in the previous case.

\[^{10}\text{shown in Appendix C.}\]
In the general case where a fraction of raw material is transformed to the final stage, but the rest is processed to level \( v \) and exported, to import intermediate goods type \( w \), the final consumption will be:

\[
A_1 = A_0 - X_v + M_w \quad (24)
\]

The equilibrium in the balance of payments requires:

\[
P(v)X_v(1-C(v)) = P(w)M_w/(1-C(w)) \quad (25)
\]

The full employment condition of capital is the following:

\[
(A_0 - X_v)\int_0^1 K(s)ds + X_v\int_0^1 K(s)ds + M_w\int_w^1 K(s)ds = K \quad (26)
\]

Substituting (25) in (24) and (26), we obtain:

\[
A_1 = A_0 - X_v + \frac{P(v)}{P(w)}(1-C(v))(1-C(w))X_v \quad (27)
\]

\[
(A_0 - X_v)\int_0^1 K(s)ds + X_v\int_0^1 K(s)ds + \frac{P(v)}{P(w)}(1-C(v))(1-C(w))X_v\int_w^1 K(s)ds = K \quad (28)
\]

Maximizing (27) subject to (28), the first order conditions are the equations (29), (30) and (31).

\[
\frac{dL}{dX_v} = -1 + \frac{P(v)}{P(w)}(1-C(v))(1-C(w)) + \lambda\left[\int_0^1 K(s)ds - \int_0^1 K(s)ds - \frac{P(v)}{P(w)}(1-C(v))(1-C(w))\int_w^1 K(s)ds\right] \quad (29)
\]

An additional unit of export reduces one unit of raw material to be converted in the final good. But one unit of export cannot finance one unit of import because the price of imported unit is higher and because of the transportation cost. This is reflected in the second term of the right hand side of (29), which is smaller than 1. This reduces the final consumption. Another effect is that it saves capital because the material, which is exported and imported afterward, is processed from 0 to \( v \) and from \( w \) to 1, instead of 0 to 1. At optimum, the sum of these two effects must be zero.

\[
\frac{dL}{dv} = [P'(v) - P'(v)C(v) - P(v)C'(v)] \frac{1-C(w)}{P(w)} X_v
\]
\[
+ \lambda \left[ -K(v) - \frac{1-C(v)}{P(w)} \int_{w} K(s) ds \left[ P'(w)(1-C(v)) - P(v)C'(v) \right] \right] X_v
\]  

(30)

The increase of \( v \) has the following effects: first, it increases the value of total export allowing a larger amount of import; second, it increases the value of exports and, as a consequence the cost of transportation, which is in proportion of the value; third, it reduces the transportation cost as a proportion of total value of export, resulting in less transportation cost; and forth, it requires more capital to transform the raw material to a higher level.

\[
\frac{dL}{dw} = - \frac{P'(w) - P'(w)C(w) + P(w)C'(w)}{P(w)^2} P(v)(1-C(v)) X_v
+ \lambda \left[ P(v) X_v (1-C(v)) \left\{ \frac{P'(w)}{P(w)^2}(1-C(w)) + \frac{C'(w)}{P(w)} \right\} \right] K(s) ds + \frac{1-C(w)}{P(w)} K(w)
\]  

(31)

The increase of \( w \) has the following effects: first, the higher price of imported goods reduces the amount of goods available; second, although it reduces the proportion of transportation cost in the value of imports, the cost of one unit of import increases because of the increase in the price of the imported goods. These two effects reduce imports aimed at final consumption and are reflected in the first term of the right hand side of (31). Third, it reduces the capital required to transform the imported intermediate good to a final product.

As usual, internal solution is obtained when the three equation are equal to zero. In an internal solution, a fraction of the raw material is exported after being processed to a certain level, and the other is processed to the final stage and consumed internally.

D. Some Implications

Without any endowment of capital, the country has only one option: export raw material and import final consumption good. With some amount of capital, which is not enough to process all its raw material, the country can either i) export raw material and import intermediate product; or ii) export intermediate good and import final consumption good; or iii) export and import intermediate good. Anyhow, it must export or import some intermediate product.

If we calculate the proportion of intermediate goods trade in total trade, this must be either 0, 1/2 or 1 in this model. This measure would not be so interesting, although not inconsistent with observed fact. If we look at the country data, we can find that the proportion of intermediate goods trade in almost all the countries is between 70 to 80 percent of its total trade.

A better measure of trade in intermediate products must reflect the level of processing of the intermediate goods traded. One implication of the model is that in the countries with an excess endowment of natural resources, the larger is the endowment of capital, the closer will be the level of processing of the exports to that of the imports.
However, the countries with excess endowment of capital will have to export products with higher level of processing and import products with lower level of processing. Therefore, an excess endowment of capital is as good a reason for trading intermediate products as an excess endowment of natural resources, and the transportation cost works in the same way in both cases. The final consumption would be equal to the value of raw material and capital at world prices less the cost of transportation.\(^\text{11}\)

From the two cases, we can conclude that the countries with excess endowment of natural resources tend to export intermediate goods with lower level of processing than the countries with excess endowment of capital, and that the more balanced is the country's endowment of capital and natural resources, the closer is the level of processing of the exports to that of the import.

III. CONCLUSION

The decision as to how much to process the raw material before exporting depends on the capital requirements for the processing and the change in transportation cost due to processing. The lower the requirements of capital and the more rapidly the transportation cost falls in the earlier stages of production, the more probable that the country will export an processed intermediate good instead of raw material. The higher the requirement of capital and the slower the fall in transportation cost in the later stages, the more probable that the country will import intermediate product instead of final consumption good. The countries with excess endowment of natural resources tend to export intermediate goods with lower level of processing than the countries with excess endowment of capital, and the more balanced is the country's endowment of capital and natural resources, the closer is the level of processing of the exports to that of the import.

For a further research, one can imagine a simple extension of the current model with more than one final product with different factor requirement ratios for the natural resources and capital. Then, the country can come up a combination of output levels for final goods that fully utilizes its domestic resources, then trade the final goods in the world market to maximize its social welfare, yielding trade in factor contents like that of Hecksher-Ohlin model. In the absence of transportation cost, such trade in the final goods would attain the social welfare maximization. When there exist transportation costs, however, trading in final goods does not necessarily the best way of trading products to balance the bias in the factor endowments. There may exist incentives to trade intermediate products to reduce the loss associated with transporting product across nations due to the same reasons described in this paper. After introducing more consumption goods that require different processing stages (intermediate products) into the model that approximate a small economy's consumption pattern (like Taiwan or Korea) together with proper measures for the country's endowments of natural resources and other factors of production, one may think about doing a calibration study to check how much of trade in

\(^{11}\) Similar to the proof in Appendix C.
intermediate products can be explained as a device of saving transportation costs for the required trade in factor contents.

APPENDIX A

The problem is to maximize \( A_1 = A_w \)
The equilibrium in the balance of trade requires:

\[
P(v)A_v(1-C(v)) = P(w)A_w/(1-C(w))
\]

As \( A_v=A_0 \) and \( A_w=A_1 \):

\[
\left[ P(1) - r \int_w^1 K(s)ds \right] A_1 = (1-C(v))(1-C(w))P(v)A_0
\]

Full employment of capital implies:

\[
A_0 \int_0^w K(s)ds + A_1 \int_w^1 K(s)ds = K
\]

Combining the two equations, we have:

\[
\left[ P(1) - r \frac{K - A_0 \int_0^1 K(s)ds}{A_1} \right] A_1 = (1-C(v))(1-C(w))P(v)A_0
\]

From the above equation, we can derive \( P(1)A_1 \):

\[
P(1)A_1 = P(0)A_0 + rK - P(v)A_v\{C(v) + C(w)(1-C(v))\}
\]

APPENDIX B

\[
A_0 \int_0^w K(s)ds + \frac{P(v)}{P(w)}A_0(1-C(v))(1-C(w))\int_w^1 K(s)ds = K
\]

With total differentiation:
It is clear that $\frac{dw}{dv} > 0$

APPENDIX C

The final consumption is:

$$A_1 = A_0 - X_v + M_w \quad (A1)$$

The equilibrium in the balance of trade requires:

$$P(v)X_v(1-C(v)) = P(w)M_w/(1-C(w)) \quad (A2)$$

Full employment of capital implies:

$$\int_{v_0}^{v_1} K(s)ds + X_v \int_{v_0}^{v_1} K(s)ds + M_w \int_{w_0}^{w_1} K(s)ds = K \quad (A3)$$

Deriving $M_w$ from (A2) and substituting it into (A1), we have:

$$P(w)A_1 = P(w)A_0 - P(w)X_v + P(v)X_v(1-C(v))(1-C(w))$$

$$\left[ P(1) - r \int_{v_0}^{v_1} K(s)ds \right]A_1$$

$$= \left[ P(0) + r \int_{v_0}^{v_1} K(s)ds \right]A_0 - \left[ P(v) + r \int_{v_0}^{v_1} K(s)ds \right]X_v + (1-C(v))(1-C(w))P(v)X_v \quad (A4)$$

From (A1) and (A3):

$$A_1 \int_{v_0}^{v_1} K(s)ds = K - A_0 \int_{v_0}^{v_1} K(s)ds + X_v \int_{v_0}^{v_1} K(s)ds$$

Substituting this into (d), we obtain:

$$P(1)A_1 = P(0)A_0 + rK - P(v)X_v\{C(v) + C(w)(1-C(v))\}$$
REFERENCES


